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双曲空间中有限个完全渐近非扩张非自映像的Δ-收敛性

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摘要：双曲空间是一类更宽泛的度量空间，论文主要通过双曲空间及完全渐近非扩张非自映像的定义，在完备一致凸双曲空间的条件下研究双曲空间中完全渐近非扩张非自映像的收敛性问题，将改进的 Ishkawa 迭代从 Banach 空间引入到双曲空间，并结合相关引理和附加条件，给出双曲空间中任意有限个完全渐近非扩张非自映像的Δ-收敛性，从而改进和推广了双曲空间的相应性质。

关 键 词：基础数学；泛函分析；不动点；完全渐近非扩张非自映像；一致利普希茨；双曲空间；Δ-收敛

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Kohlenbach Ulrich 在 2005 年于文献[1]创立了双曲空间，是一类更宽泛的度量空间.

文中是将改进的 Ishkawa 迭代从 Banach 空间引入到了双曲空间，讨论双曲空间中任意有限个完全渐近非扩张非自映像的Δ-收敛性问题.

1 预备知识

定义 1[1] 如果(X,d)是一个度量空间，$W:X\times X\times[0,1]\to X$ 是一个映射，满足：

(i) $\forall x,y,z\in X,\forall\lambda\in[0,1],d(z,W(x,y,\lambda))\le(1-\lambda)d(z,x)+\lambda d(z,y)$；

(ii) $\forall x,y\in X,\forall\lambda_1,\lambda_2\in[0,1],d(W(x,y,\lambda_1),W(x,y,\lambda_2))=|\lambda_1-\lambda_2|d(x,y)$；

(iii) $\forall x,y\in X,\forall\lambda\in[0,1],W(x,y,\lambda)=W(y,x,(1-\lambda))$；

(iiii) $\forall x,y,z,w\in X,\forall\lambda\in[0,1],d(W(x,z,\lambda),W(y,w,\lambda))\le(1-\lambda)d(x,y)+\lambda d(z,w)$.

那么称(X,d,W)为双曲空间.

定义 2[2-3] (X,d,W)为一双曲空间，如果对 $\forall u,x,y\in X,r>0$，且 $\varepsilon\in(0,2]$，存在 $\delta\in(0,1]$，使得

$$d(W(x,y,\frac{1}{2}),u)\le(1-\delta)r$$

其中

$$d(x,u)\le r,d(y,u)\le r,d(x,y)\ge\varepsilon r$$

那么称双曲空间是一致凸的.

定义 3[4] 映射 $\eta:(0,\infty)\times(0,2]\to(0,1]$，如果对于给定的 $r>0$，使得

$$\delta=\eta(r,\varepsilon)$$

那么称 η 为一致凸单元；如果对给定的 $\varepsilon\in(0,2]$，有：

$$\eta(r_2,\varepsilon)\le\eta(r_1,\varepsilon),\forall r_2\ge r_1>0$$

那么称 η 是单调的.

定义 4[5] 如果$\{x_n\}$为双曲空间 X 中的有界序列，$x\in X$，定义如下概念：

$$r(x,\{x_n\})=\lim_{n\to\infty}\sup d(x,x_n);$$

$\{x_n\}$的渐近半径 $r(\{x_n\})$：

$$r(x,\{x_n\})=\inf\{r(x,\{x_n\}):x\in X\};$$

$\{x_n\}$的渐近半径 $r_C(\{x_n\})$，$C\subset X$：

$$r_C(\{x_n\})=\inf\{r(x,\{x_n\}):x\in C\};$$

$\{x_n\}$的渐近中心 $A(\{x_n\})$：

$$A(\{x_n\})=\{x\in X:r(x,\{x_n\})=r(\{x_n\})\};$$

$\{x_n\}$的渐近中心 $A_C(\{x_n\})$，$C\subset X$：

$$A_C(\{x_n\})=\{x\in C:r(x,\{x_n\})=r_C(\{x_n\})\};$$

设序列$\{u_n\}$是$\{x_n\}$的任一子序列，如果 $x\in X$ 是$\{u_n\}$唯一的渐近中心，那么称序列$\{x_n\}$Δ-收敛于 x.

定义 5[6-7] (X,d)为一度量空间，C 是 X 的非空子集，如果存在连续映射 $P:X\to C$，使得 $Px=x,\forall x\in C$. 那么称 C 为 X 的一个限制；如果映射 $P:X\to C$，使得 $P^2=P$，那么 P 称为限制.

定义 6[8-9] 非自映像 $T:C\to X$，如果存在非负序列$\{v_n\},\{\mu_n\},v_n\to0,u_n\to0$ 和严格增的连续函数 $\zeta:[0,\infty)\to[0,\infty)$且 $\zeta(0)=0$，使得

$$d(T(PT)^{n-1}x,T(PT)^{n-1}y)\le d(x,y)+v_n\zeta(d(x,y))+\mu_n,\forall n\ge1,x,y\in C$$

其中 P 是 $X\to C$ 的非扩张限制，那么称 $T:C\to X$ 为$(\{v_n\},\{\mu_n\},\zeta)$-完全渐近非扩张非自映像.

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定义 7[10-11] $T:C\to X$,如果存在常数 $L>0$,使得

$$d(T(PT)^{n-1}x,T(PT)^{n-1}y)\le Ld(x,y),\ \forall n\ge 1,x,y\in C$$

那么称 $T:C\to X$ 是一致 *L-Lipschitzian* 的.

引理 1[12-13] (X,d,W) 为完备一致凸双曲空间,η 为一致凸的单调调系数,C 是 X 的非空闭凸子集. 那么 X 中的任一有界序列 $\{x_n\}$ 关于 C 有唯一的渐近中心.

引理 2[14-15] (X,d,W) 为一致凸双曲空间,η 为一致凸的单调系数,$x\in X$,序列 $\{\alpha_n\}$ 在 $[a,b]$ 中,其中 $a,b\in(0,1)$. 如果序列 $\{x_n\}$,$\{y_n\}$ 在 X 中,使得

$$\limsup_{n\to\infty} d(x_n,x)\le c$$

$$\limsup_{n\to\infty} d(y_n,x)\le c$$

$$\lim_{n\to\infty} d(W(x_n,y_n,\alpha_n),x)=c$$

那么

$$\lim_{n\to\infty} d(x_n,y_n)=0\quad(c\ge 0).$$

引理 3[16] $\{a_n\}$,$\{b_n\}$ 和 $\{c_n\}$ 为非负序列,使得 $a_{n+1}\le(1+b_n)a_n+c_n$,$\forall n\ge 1$. 如果满足 $\sum\limits_{n=1}^{\infty}b_n<\infty$ 和 $\sum\limits_{n=1}^{\infty}c_n<\infty$,那么 $\lim\limits_{n\to\infty}a_n$ 存在.

2 主要结果

定理 1 X 为完备一致凸双曲空间,η 为一致凸的单调系数,C 是 X 的非空闭凸子集. 令 $T_i:C\to X$,$(i=1,2,\cdots,m)$,是一致 *L-Lipschitzian* 和 $(\{v_{in}\},\{\mu_{in}\},\zeta_i)$-完全渐近非扩张非自映像. 对于任意给定的 $x_1\in C$,$\{x_n\}$ 定义如下:

$$\begin{cases} x_1\in C,\\ x_{n+1}=PW(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),m=1,n\ge 1;\\ x_1\in C,\\ x_{n+1}=PW(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),\\ y_{1n}=PW(x_n,T_2(PT_2)^{n-1}y_{2n},\alpha_{2n}),\\ y_{2n}=PW(x_n,T_3(PT_3)^{n-1}y_{3n},\alpha_{3n}),\\ \cdots\\ y_{(m-2)n}=PW(x_n,T_{m-1}(PT_{m-1})^{n-1}y_{(m-1)n},\alpha_{(m-1)n}),\\ y_{(m-1)n}=PW(x_n,T_m(PT_m)^{n-1}x_n,\alpha_{mn}),m\ge 2,n\ge 1. \end{cases}\tag{1}$$

其中 P 是 $X\to C$ 的非扩张限制. 假定$\mathcal{F}=:\cap_{i=1}^{m}F(T_i)\ne\phi$. 且满足如下条件:

(i) $\sum_{n=1}^{\infty}v_{in}<\infty$,$\sum_{n=1}^{\infty}\mu_{in}<\infty$;

(ii) 存在常数 $a,b\in(0,1)$,使得 $\{\alpha_{in}\}$,$\{\beta_{in}\}\subset[a,b]$;

(iii) 存在常数 $M_i,M^*a_i>0$,使得当 $r_i>M_i$ 时,有 $\zeta_i(r_i)\le M_i^*r_i$.

其中,$i=1,2,\cdots,m$.

那么,由式(1)定义的序列 $\{x_n\}$ Δ-收敛于$\mathcal{F}$中一点.

证明:将证明分为 3 步完成.

第一步:证明 $\lim\limits_{n\to\infty}d(x_n,p)$ 存在,$p\in\mathcal{F}$.

由于 ζ_i 是严格增的连续函数,所以当 $r_i\le M_i$ 时,有 $\zeta_i(r_i)\le\zeta_i(M_i)$;又由(iii)知,

当 $r_i>M_i$ 时,有 $\zeta_i(r_i)\le M_i^*r_i$. 其中,$i=1,2,\cdots,m$.

因此

$$\zeta_i(d(x_n,p))\le\zeta_i(M_i)+{}_{*\,i}d(x_n,p)$$

当 $m=1$ 时,$d(x_{n+1},p)=d(PW(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),p)$

$\le d(W(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),p)$

$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}d(T_1(PT_1)^{n-1}x_n,p)$

$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}[d(x_n,p)+v_{1n}\zeta_1(d(x_n,p))+\mu_{1n}]$

$\le d(x_n,p)+\alpha_{1n}v_{1n}\zeta_1(M_1)+\alpha_{1n}v_{1n}M_1^*d(x_n,p)+\alpha_{1n}\mu_{1n}$

$\le(1+v_{1n}M_1^*)d(x_n,p)+v_{1n}\zeta_1(M_1)+\mu_{1n}$

所以存在 $Q_1>0$,使得

$$d(x_{n+1},p)\le(1+v_{1n}Q_1)d(x_n,p)+(v_{1n}+\mu_{1n})Q_1$$

$$\sum_{n=1}^{\infty}v_{1n}Q_1<\infty,\ \sum_{n=1}^{\infty}(v_{1n}+\mu_{1n})Q_1<\infty$$

由引理 3 可知,$\lim\limits_{n\to\infty}d(x_n,p)$ 存在,当 $m=1$ 时成立.

当 $m=2$ 时,$\begin{cases} x_{n+1}=PW(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n})\\ y_{1n}=PW(x_n,T_2(PT_2)^{n-1}x_n,\alpha_{2n})\end{cases}$

$d(y_{1n},p)=d(PW(x_n,T_2(PT_2)^{n-1}x_n,\alpha_{2n}),p)$

$\le d(W(x_n,T_2(PT_2)^{n-1}x_n,\alpha_{2n}),p)$

$\le(1-\alpha_{2n})d(x_n,p)+\alpha_{2n}d(T_2(PT_2)^{n-1}x_n,p)$

$\le(1-\alpha_{2n})d(x_n,p)+\alpha_{2n}[d(x_n,p)+v_{2n}\zeta_2(d(x_n,p))+\mu_{2n}]$

$\le d(x_n,p)+\alpha_{2n}v_{2n}v_{2n}\zeta_2(M_2)+\alpha_{2n}v_{2n}M_2^*d(x_n,p)+\alpha_{2n}\mu_{2n}$

$\le(1+v_{2n}M_2^*)d(x_n,p)+v_{2n}\zeta_2(M_2)+\mu_{2n}$ (2)

$d(x_{n+1},p)=d(PW(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),p)$

$\le d(W(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),p)$

$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}d(T_1(PT_1)^{n-1}y_{1n},p)$

$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}[d(y_{1n},p)+v_{1n}\zeta_1(d(y_{1n},p))+\mu_{1n}]$

$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}\{d(y_{1n},p)+v_{1n}[\zeta_1(M_1)+M_1^*d(y_{1n},p)]+\mu_{1n}\}$

$\le(1-\alpha_{1n})d(x_n,p)+d(y_{1n},p)+v_{1n}\zeta_1(M_1)+v_{1n}M_1^*d(y_{1n},p)+\mu_{1n}$

$=(1-\alpha_{1n})d(x_n,p)+(1+v_{1n}M_1^*)d(y_{1n},p)+v_{1n}\zeta_1(M_1)+\mu_{1n}]$ (3)

将式(2)代入式(3)有:

$d(x_{n+1},p)\le(1-\alpha_{1n})d(x_n,p)+(1+v_{1n}M_1^*)\{(1+v_{2n}M_2^*)d(x_n,p)+v_{2n}\zeta_2(M_2)+\mu_{2n}\}+v_{1n}\zeta_1(M_1)+\mu_{1n}$

$=(1-\alpha_{1n})d(x_n,p)+(1+v_{1n}M_1^*)(1+v_{2n}M_2^*)d(x_n,p)+(1+v_{1n}M_1^*)(v_{2n}\zeta_2(M_2)+\mu_{2n})+v_{1n}\zeta_1$

$$(M_1)+\mu_{1n}$$
$$=(1+1-\alpha_{1n}+v_{1n}M_1^*+v_{2n}M_2^*+v_{1n}M_1^*v_{2n}M_2^*)d(x_n,p)+v_{1n}\zeta_1(M_1)+v_{2n}\zeta_2(M_2)+\mu_{1n}+v_{1n}M_1^*v_{2n}\zeta_2(M_2)+\mu_{2n}+v_{1n}\mu_{2n}M_1^*$$

因此,存在 $Q_2>0$,使得

$$d(x_{n+1},p)\le(1+(v_{1n}+v_{2n})Q_2)d(x_n,p)+(v_{1n}+v_{2n}+\mu_{1n}+\mu_{2n})Q_2$$

$$\sum_{n=1}^{\infty}(v_{1n}+v_{2n})Q_2<\infty,\sum_{n=1}^{\infty}(v_{1n}+v_{2n}+\mu_{1n}+\mu_{2n})Q_2<\infty$$

由引理 3 可知,$\lim\limits_{n\to\infty}d(x_n,p)$存在,当 $m=2$ 时成立.

用相同的方法一直做下去,以此类推,能够得到存在 $Q>0$,使得:

$$d(x_{n+1},p)\le(1+\sum_{i=1}^{m}v_{in}Q)d(x_n,p)+\sum_{i=1}^{m}(v_{in}+\mu_{in})Q$$

成立.

其中

$$\sum_{i=1}^{m}v_{in}Q<\infty,\sum_{i=1}^{m}(v_{in}+\mu_{in})Q<\infty$$

因此,由引理 3,$\lim\limits_{n\to\infty}d(x_n,p)$存在,$m\ge1$,$m\in Z^+$.

第二步:证明 $d(x_n,T_ix_n)\to0\quad(n\to\infty)$,$i=1,2,\cdots,m$.

由第一步知$\lim\limits_{n\to\infty}d(x_n,p)$存在,不防设$\lim\limits_{n\to\infty}d(x_n,p)=c>0$.

当 $m=1$ 时,显然

$$\limsup_{n\to\infty}d(x_n,p)\le c \tag{4}$$

$$d(T_1(PT_1)^{n-1}x_n,p)\le d(x_n,p)+v_{1n}\zeta_1(d(x_n,p))+\mu_{1n}\to c\quad(n\to\infty)$$

因此

$$\limsup_{n\to\infty}d(T_1(PT_1)^{n-1}x_n,p)\le c \tag{5}$$

又由

$$c=d(x_{n+1},p)=d(PW(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),p)$$
$$\le d(W(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),p)\to c\quad(c\to\infty)$$

有

$$\lim_{n\to\infty}d(W(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),p)=c \tag{6}$$

由(4)、(5)、(6)式以及引理 2,有

$$\lim_{n\to\infty}d(x_n,T_1(PT_1)^{n-1}x_n)=0$$

而

$$d(x_{n+1},x_n)=d(PW(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),x_n)$$
$$\le d(W(x_n,T_1(PT_1)^{n-1}x_n,\alpha_{1n}),x_n)$$
$$\le\alpha_{1n}d(T_1(PT_1)^{n-1}x_n,x_n)\to0\quad(n\to\infty)$$

所以

$$d(x_n,T_1x_n)\le d(x_n,x_{n+1}+d(T_1(PT_1)^nx_{n+1},T_1(PT_1)^nx_n)+d(x_{n+1},T_1(PT_1)^nx_{n+1})+d(T_1(PT_1)^nx_n,T_1x_n)$$
$$\le(1+L)d(x_{n+1},x_n)+d(x_{n+1},T_1(PT_1)^nx_{n+1})+Ld(x_n,T_1(PT_1)^{n-1}x_n)\to0\quad(n\to\infty)$$

所以

$$d(x_n,T_ix_n)\to0\quad(n\to\infty),i=1\text{ 成立}.$$

当 $m=2$ 时,$\begin{cases}x_{n+1}=PW(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n})\\ y_{1n}=PW(x_n,T_2(PT_2)^{n-1}x_n,\alpha_{2n})\end{cases}$

显然

$$\limsup_{n\to\infty}d(x_n,p)\le c \tag{7}$$

$$d(T_2(PT_2)^{n-1}x_,p)\le d(x_n,p)+v_{2n}\zeta_2(d(x_n,p))+\mu_{2n}\to c\quad(n\to\infty)$$

因此

$$\limsup_{n\to\infty}d(T_2(PT_2)^{n-1}x_n,p)\le c \tag{8}$$

由(2)式知

$$d(y_{1n},p)\le(1+v_{2n}M_2^*)d(x_n,p)+v_{2n}\zeta_2(M_2)+\mu_{2n}\to c\quad(c\to\infty)$$

而

$$d(x_{n+1},p)=d(PW(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),p)$$
$$\le d(W(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),p)$$
$$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}d(T_1(PT_1)^{n-1}y_{1n},p)$$
$$\le(1-\alpha_{1n})d(x_n,p)+\alpha_{1n}[d(y_{1n},p)+v_{1n}\zeta_1(d(y_{1n},p))+\mu_{1n}]$$

有:

$$d(x_{n+1},p)-(1-\alpha_{1n})d(x_n,p)\le\alpha_{1n}[d(y_{1n},p)+v_{1n}\zeta_1(d(y_{1n},p))+\mu_{1n}]$$

从而

$$\lim_{n\to\infty}\alpha_{1n}d(x_n,p)\le\lim_{n\to\infty}\alpha_{1n}d(y_{1n},p)$$

因此

$$\lim_{n\to\infty}d(y_{1n},p)=c$$

即

$$\lim_{n\to\infty}d(W(x_n,T_2(PT_2)^{n-1}x_n,\alpha_{2n}),p)=c \tag{9}$$

由引理 2 及(7)、(8)、(9)式,可得

$$d(y_{1n},x_n)=d(PW(x_n,T_2(PT_2)^{n-1}x_n,\alpha_{2n}),x_n)$$
$$\le d(x_n,T_2(PT_2)^{n-1}x_n)\to0\quad(n\to\infty) \tag{10}$$

又

$$\limsup_{n\to\infty}d(x_n,p)\le c \tag{11}$$

$$d(T_1(PT_1)^{n-1}y_{1n},p)\le d(y_{1n},p)+v_{1n}\zeta_1(d(y_{1n},p))+\mu_{1n}\to c\quad(n\to\infty)$$

因此

$$\limsup_{n\to\infty}d(T_1(PT_1)^{n-1}y_{1n},p)\le c \tag{12}$$

$$c=\lim_{n\to\infty}d(x_{n+1},p)=\lim_{n\to\infty}d(PW(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),p)\le\lim_{n\to\infty}d(W(x_n,T_1(PT_1)^{n-1}y_{1n},\alpha_{1n}),p)\le c$$

因此

$$\lim_{n\to\infty} d(W(x_n, T_1(PT_1)^{n-1}y_{1n}, \alpha_{1n}), p) = c \quad (13)$$

由(11)、(12)、(13)式以及引理2,可得

$$\lim_{n\to\infty} d(x_n, T_1(PT_1)^{n-1}y_{1n}) = 0 \quad (14)$$

由(10)、(14)式,有

$$\begin{aligned} d(x_n, T_1(PT_1)^{n-1}x_n) &\le d(x_n, T_1(PT_1)^{n-1}y_{1n}) + d(T_1(PT_1)^{n-1}y_{1n}, T_1(PT_1)^{n-1}x_n) \\ &\le d(x_n, T_1(PT_1)^{n-1}y_{1n}) + Ld(y_{1n}, x_n) \to 0 \quad (n\to\infty) \end{aligned} \quad (15)$$

因此,由(10)、(15)式,可得

$$\lim_{n\to\infty} d(x_n, T_i(PT_i)^{n-1}x_n) = 0, \quad i=1,2.$$

再

$$\begin{aligned} d(x_{n+1}, x_n) &= d(PW(x_n, T_1(PT_1)^{n-1}y_{1n}, \alpha_{1n}), x_n) \\ &\le d(W(x_n, T_1(PT_1)^{n-1}y_{1n}, \alpha_{1n}), x_n) \\ &\le \alpha_{1n} d(T_1(PT_1)^{n-1}y_{1n}, x_n) \to 0 \quad (n\to\infty) \end{aligned} \quad (16)$$

$$d(x_n, T_ix_n) \le (1+L)d(x_{n+1}, x_n) + d(x_{n+1}, T_i(PT_i)^n x_{n+1}) + Ld(x_n, T_i(PT_i)^{n-1}x_n) \to 0 \quad (n\to\infty), i=1,2.$$

同理,由 T_i 的连续性,用相同的方法,以此类推,能够得到

$\lim_{n\to\infty} d(y_{in}, p) = c, i=1,2,\cdots,(m-1)$;

$\lim_{n\to\infty} d(x_n, T_i(PT_i)^{n-1}y_{in}) = 0, i=1,2,\cdots,(m-1)$;

$\lim_{n\to\infty} d(x_n, T_m(PT_m)^{n-1}x_n) = 0$;

$\lim_{n\to\infty} d(y_{in}, x_n) = 0, i=1,2,\cdots,(m-1)$;

$\lim_{n\to\infty} d(x_n, T_i(PT_i)^{n-1}x_n) = 0, i=1,2,\cdots,(m-1)$;

$\lim_{n\to\infty} d(x_{n+1}, x_n) = 0.$

得

$$\lim_{n\to\infty} d(x_n, T_ix_n) = 0, i=1,2,3,\cdots,m.$$

第三步:现在能够证明序列$\{x_n\}$是Δ-收敛的. 由于$\{x_n\}$是有界的,那么他有唯一的渐近中心 $A_C(\{x_n\}) = \{x^*\}$. 令$\{u_n\}$为$\{x_n\}$的任一子序列,其渐近中心为

$$A_C(\{u_n\}) = \{u\}.$$

由于$\lim_{n\to\infty} d(x_n, T_ix_n) = 0, i=1,2,\cdots,m$. 那么有 $u \in F$. 又

$$r(\{x_n\}, u) \ge r(\{x_n\}, x^*) = r(\{u_n\}, x^*) \ge r(\{u_n\}, u) \ge r(\{x_n\}, u)$$

有 $x^* = u$. 这说明 x^* 是序列$\{x_n\}$的任一子序列$\{u_n\}$的唯一渐近中心,也就是说序列$\{x_n\}$是 Δ-收敛于 $x^* \in F$. 证明完毕.

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Δ-Convergence Character for Finite Families of Total Asymptotically Nonexpansive Nonself Mappings in Hyperbolic Spaces

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Abstract: The hyperbolic spaces covers all normed liner spaces. This paper mainly applied the concept of hyperbolic spaces and total asymptotically nonexpansive nonself mappings , discuss the convergence character of total asymptotically nonexpansive nonself mappings in complete uniformly convex hyperbolic spaces. And translation the Ishkawa iterative from Banach spaces to hyperbolic spaces. Using the relevant lemma and auxiliary conditions, we prove the Δ-convergence character for finite families of total asymptotically nonexpansive nonself mappings in hyperbolicspaces. Our results extend some results in the literature.

Key words: fundamental mathematics; functional analysis; fixed point; total asymptotically nonexpansive nonself mapping; uniformly L-Lipschitzian; hyperbolicspace; Δ-convergence