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Convergence of Modified Proximal Point Algorithms for Hybrid Pair of Nonexpansive Mappings in $CAT(0)$ Spaces

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Abstract: In this paper, we propose a modied proximal point algorithm for finding a common element of the set of fixed points of a single-valued nonexpansive mapping, the set of fixed points of a multivalued nonexpansive mapping, and the set of minimizers of convex and lower semicontinuous functions. We obtain the strong convergence to a common element of three sets in $CAT(0)$ spaces.

Keywords: $CAT(0)$ space; proximal point algorithm; single-valued nonexpansive mapping; multi-valued nonexpansive mapping; resolvent identity

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1 Introduction

A metric space (X,d) is a $CAT(0)$ space[1] if it is geodesically connected and each geodesic triangle is at least as 'thin' as its comparison triangle in R^2. Let D be a nonempty closed subset of a $CAT(0)$ space X, and let T: $D\to D$ be a mapping. The set of fixed point of T is denoted by $F(T)$, that is, $F(T)=\{x\in D: x=Tx\}$.

The useful inequality of $CAT(0)$ space is (CN) inequality[2], that is, if z, x, y are some points in a $CAT(0)$ space and if $\frac{x\oplus y}{2}$ is the midpoint of geodesic segment $[x,y]$, then the $CAT(0)$ inequality implies

$$d^2(z,\frac{x\oplus y}{2})\leqslant\frac{1}{2}d^2(z,x)+\frac{1}{2}d^2(z,y)-\frac{1}{4}d^2(x,y), \tag{CN}$$

which is equivalent to the following[3]

$$d^2(z,\lambda x\oplus(1-\lambda)y)\leqslant\lambda d^2(z,x)+(1-\lambda)d^2(z,y)-\lambda(1-\lambda)d^2(x,y), \quad (CN^*)$$

for any $\lambda\in[0,1]$, where $\lambda x\oplus(1-\lambda)y$ denotes a unique point in $[x,y]$. Moreover, if X is a $CAT(0)$ space and $x,y\in X$, then for any $\lambda\in[0,1]$, there exists a unique point $\lambda x\oplus(1-\lambda)y\in[x,y]$ such that

$$d(z,\lambda x\oplus(1-\lambda)y)\leqslant\lambda d(z,x)+(1-\lambda)d(z,y), \tag{1}$$

for any $z\in X$.

In the past, there has been many iterative methods that are constructed and proposed to find approximate fixed points of nonlinear mappings. The S-type iterative methods is defined as follows:

$$\begin{cases} y_n=(1-\alpha_n)x_n+\alpha_n Tx_n, \\ x_{n+1}=(1-\beta_n)Tx_n+\beta_n Ty_n, \end{cases}$$

for each $n\in N$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $(0,1)$.

The proximal point algorithm, which was first introduced by Martinet, is known for its theoretically nice convergence properties.

Bacak[5] introduced proximal point algorithm into geodesic metric spaces of nonpositive curvature, that is, $CAT(0)$ spaces. For any initial point x_1 in a $CAT(0)$ space X, a sequence $\{x_n\}$ generated by

$$x_{n+1}=\underset{y\in X}{\operatorname{argmin}}[f(y)+\frac{1}{2\lambda_n}\mathrm{d}^2(y,x_n)],$$

where $\lambda_n>0$ for all $n\in N$.

For all $\lambda>0$, define the Moreau-Yosida[6] resolvent of f in a complete $CAT(0)$ space X as follows:

$$J_\lambda(x)=\underset{y\in C}{\operatorname{argmin}}[f(y)+\frac{1}{2\lambda}d^2(y,x)].$$

Let $f: X\to(-\infty,+\infty)$ be a proper convex and lower semi-continuous function. The set $F(J_\lambda)$ of fixed points of the resolvent associated with f coincides wifh the set $\underset{y\in X}{\operatorname{argmin}} f(y)$ of minimizers of f, which can be found in[7]. Also for any $\lambda>0$, the resolvent J_λ of f is nonexpansive[8].

In 2017, Suthep Suantai[9] have put forward the manner as follows

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$$\begin{cases} z_n = \underset{y \in C}{\operatorname{argmin}}\left[f(y) + \dfrac{1}{2\lambda_n} d^2(y, x_n) \right], \\ y_n = \beta_n z_n \oplus (1-\beta_n) w_n, w_n \in S z_n, \\ x_{n+1} = \alpha_n x_n \oplus (1-\alpha_n) T z_n, \forall n \in N, \end{cases}$$

where T is a single-valued nonexpansive mapping, S is a multi-valued nonex-pansive mapping. $\{\lambda_n\}$ is a sequence such that $\lambda_n \geqslant \lambda > 0$ for all $n \geqslant 1$ and some λ.

Stimulated and inspired by the work of the mathematics researchers, in this paper, we use S-type iteration methods and come up with a new modified algorithm that is from [9]. Research its convergence, some results that we obtained improved and extended the results of [9].

2 Preliminaries

This section collects some lemmas, definitions, which will be used in our main results in the next section.

Definition 1[10] Let $CB(D)$, $CC(D)$ and $KC(D)$ denote the families of nonempty closed bounded subsets, closed convex subsets and compact convex subsets of D, respectively. The Pompeiu-Hausdorff distance on $CB(D)$ is defined by

$H(A,B) = \max\{\sup_{x \in A} \operatorname{dist}(x,B), \sup_{y \in B} \operatorname{dist}(y,A)\}$ for $A, B \in CB(D)$,

where $\operatorname{dist}(x,D) = \inf\{d(x,y): y \in D\}$ is the distance from a point x to a subset D. Let S be a multi-valued mapping of D into $CB(D)$.

An element $x \in D$ is called a fixed point of S if $x \in Sx$. The set of all fixed points of S is denoted by $F(S)$, that is, $F(S) = \{x \in D: x \in Sx\}$.

Definition 2[9] A single-valued mapping $T: D \to D$ is said to be

(1) nonexpansive if $d(Tx, Ty) \leqslant d(x,y)$ for all $x, y \in D$;

(2) semi-compact if for any sequence $\{x_n\}$ in D such that

$$\lim_{n\to\infty} d(x_n, Tx_n) = 0,$$

there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $\{x_{n_i}\}$ converges strongly to a point p in D.

Definition 3[9] A multi-valued mapping $S: D \to CB(D)$ is said to be

(1) nonexpansive if $H(Sx, Sy) \leqslant d(x,y)$ for all $x, y \in D$;

(2) hemi-compact if for any sequence $\{x_n\}$ in D such that

$$\lim_{n\to\infty} \operatorname{dist}(x_n, Sx_n) = 0,$$

there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to a point p in D.

Lemma 1[8] Let (X,d) be a complete $CAT(0)$ space and $f: X \to (-\infty, +\infty)$ be a proper convex and lower semi-continuous function. Then the following identity holds:

$$J_\lambda x = J_\mu\left(\frac{\lambda-\mu}{\lambda} J_\lambda x \oplus \frac{\mu}{\lambda} x\right), \forall x \in X, \lambda > \mu > 0.$$

Lemma 2[11] Let (X,d) be a complete $CAT(0)$ space and $f: X \to (-\infty, +\infty)$ be a proper convex and lower semi-continuous function. Then, for all $x, y \in X$ and $\lambda > 0$, the following inequality holds:

$$\frac{1}{2\lambda} d^2(J_\lambda x, y) - \frac{1}{2\lambda} d^2(x,y) + \frac{1}{2\lambda} d^2(x, J_\lambda x) + f(J_\lambda x) \leqslant f(y),$$

where J_λ is a Moreau-Yosida resolvent of f.

3 Main results

In this part, we go to prove our main theorems.

Theorem 1 Suppose that the following conditions are satisfied:

(1) Let D be a nonempty closed convex subset of a complete $CAT(0)$ space X;

(2) Let $T: D \to D$ be a single-valued nonexpansive mapping, $S: D \to CB(D)$ be a multi-valued nonexpansive mapping, and $f: D \to (-\infty, +\infty)$ be a proper convex and lower semi-continuous function;

(3) Suppose that $\Omega = F(T) \cap F(S) \cap \underset{y \in D}{\operatorname{argmin}} f(y)$ is nonempty and $S_q = \{q\}$ for all $q \in \Omega$;

(4) Let $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ be sequences in $(0,1)$ with $0 < a \leqslant \alpha_n, \beta_n, \gamma_n \leqslant b < 1$ for all $n \in N$ and for some a, b are positive constants in $(0,1)$, and $\{\lambda_n\}$ is a sequence such that $\lambda_n \geqslant \lambda > 0$ for all $n \in N$ and some λ;

(5) Suppose that J_λ is semi-compact or T is semi-compact or S is hemicompact.

For $x_1 \in D$, the sequence $\{x_n\}$ generated by the algorithm as follows:

$$\begin{cases} z_n = \underset{y \in D}{\operatorname{argmin}}\left[f(y) + \dfrac{1}{2\lambda_n} d^2(y, x_n) \right], \\ t_n = \gamma_n z_n \oplus (1-\gamma_n) w_n, w_n \in S z_n, \\ y_n = \beta_n x_n \oplus (1-\beta_n) T t_n, \\ x_{n+1} = \alpha_n T x_n \oplus (1-\alpha_n) T y_n, \forall n \in N, \end{cases}$$

then the sequence $\{x_n\}$ converges strongly to a point in Ω.

Proof. This proof will be divided into five steps.

(i) Let $q \in \Omega$. Then we have $Tq = q \in Sq$ and $f(q) \leqslant f(y)$, for all $y \in D$. It follows that

$$f(q)+\frac{1}{2\lambda_n}d^2(q,q)\leqslant f(y)+\frac{1}{2\lambda_n}d^2(y,q),\ \forall y\in D.$$

Hence, $q=J_{\lambda_n}q$ for all $n\in N$. Since $z_n=J_{\lambda_n}x_n$, it follows by nonexpansiveness of J_{λ_n} that

$$d(z_n,q)=d(J_{\lambda_n}x_n,J_{\lambda_n}q)\leqslant d(x_n,q). \tag{3}$$

For $q\in\Omega$, by virtue of $Sq=\{q\}$ and (1), it shows that

$$\begin{aligned}d(t_n,q)=d(\gamma_n z_n)\oplus(1-\gamma_n)w_n,q\leqslant\\ \gamma_n d(z_n,q)+(1-\gamma_n)d(w_n,q)\leqslant\\ \gamma_n d(z_n,q)+(1-\gamma_n)\operatorname{dist}(q,Sz_n)\leqslant\\ \gamma_n d(z_n,q)+(1-\gamma_n)H(Sq,Sz_n)\leqslant\\ d(z_n,q)\leqslant\\ d(x_n,q).\end{aligned} \tag{4}$$

By (4), we have

$$\begin{aligned}d(y_n,q)=d(\beta_n x_n\oplus(1-\beta_n)Tt_n,q)\leqslant\\ \beta_n d(x_n,q)+(1-\beta_n)d(Tt_n,q)\leqslant\\ \beta_n d(x_n,q)+(1-\beta_n)d(t_n,q)\leqslant\\ \beta_n d(x_n,q)+(1-\beta_n)d(x_n,q)=\\ d(x_n,q)\end{aligned} \tag{5}$$

and we get

$$\begin{aligned}d(x_{n+1},q)=d(\alpha_n Tx_n\oplus(1-\alpha_n)Ty_n,q)\leqslant\\ \alpha_n d(Tx_n,q)+(1-\alpha_n)d(Ty_n,q)\leqslant\\ \alpha_n d(x_n,q)+(1-\alpha_n)d(y_n,q)\leqslant\\ d(x_n,q).\end{aligned} \tag{6}$$

Therefore, by (6), we obtain that the sequence $\{d(x_n,q)\}$ is decreasing and bounded. So, the limit $\lim\limits_{n\to\infty}d(x_n,q)$ exists for all $q\in\Omega$.

(ii) Let

$$\lim_{n\to\infty}d(x_n,q)=c\geqslant 0. \tag{7}$$

By lemma 2, we have

$$\frac{1}{2\lambda_n}d^2(z_n,q)-\frac{1}{2\lambda_n}d^2(x_n,q)+\frac{1}{2\lambda_n}d^2(z_n,x_n)\leqslant f(q)-f(z_n).$$

Since $f(q)\leqslant f(z_n)$ for all $n\in N$, we get

$$d^2(z_n,x_n)\leqslant d^2(x_n,q)-d^2(z_n,q) \tag{8}$$

By (3), we obtain

$$\limsup_{n\to\infty}d(z_n,q)\leqslant\limsup_{n\to\infty}d(x_n,q)=c. \tag{9}$$

Let $\alpha_n=1-(1-\alpha_n)$ and by (6), we have

$$(1-\alpha_n)d(x_n,q)\leqslant d(x_n,q)-d(x_{n+1},q)+(1-\alpha_n)d(y_n,q)$$

and

$$d(x_n,q)\leqslant\frac{1}{1-\alpha_n}[d(x_n,q)-d(x_{n+1},q)]+d(y_n,q)\leqslant \frac{1}{1-b}[d(x_n,q)-d(x_{n+1},q)]+d(y_n,q).$$

This implies that

$$c=\lim_{n\to\infty}\inf d(x_n,q)\leqslant\lim_{n\to\infty}\inf\{\frac{1}{1-b}[d(x_n,q)-d(x_{n+1},q)]+d(y_n,q)\}\leqslant \lim_{n\to\infty}\inf d(y_n,q),$$

that is,

$$c=\lim_{n\to\infty}\inf d(x_n,q)\leqslant\lim_{n\to\infty}\inf d(y_n,q).$$

Similarly, by (5) and we get

$$\lim_{n\to\infty}\sup d(y_n,p)\leqslant\lim_{n\to\infty}\sup d(x_n,q)=c.$$

Therefore,

$$\lim_{n\to\infty}d(y_n,q)=c. \tag{10}$$

Furthermore, by the inequality (5), we also have

$$d(y_n,q)\leqslant\beta_n d(x_n,q)+(1-\beta_n)d(t_n,q)$$

and let $\beta_n=1-(1-\beta_n)$, similarly we have

$$(1-\beta_n)d(x_n,q)\leqslant d(x_n,q)-d(y_n,q)+(1-\beta_n)d(t_n,q),$$

which can be rewritten as

$$d(x_n,q)\leqslant\frac{1}{1-\beta_n}\{d(x_n,q)-d(y_n,q)+(1-\beta_n)d(t_n,q)\}\leqslant \frac{1}{1-b}[d(x_n,q)-d(y_n,q)]+d(t_n,q).$$

It implies that

$$\lim_{n\to\infty}\inf d(x_n,q)\leqslant\lim_{n\to\infty}\inf\{\frac{1}{1-b}[d(x_n,q)-d(y_n,q)]+d(t_n,q)\}.$$

This together with (7) and (10), we obtain

$$\lim_{n\to\infty}\inf d(x_n,q)=c\leqslant\lim_{n\to\infty}\inf d(t_n,q). \tag{11}$$

Also from (4), we have

$$\lim_{n\to\infty}\sup d(t_n,q)\leqslant\lim_{n\to\infty}\sup d(x_n,q)=c.$$

By (11), it shows that

$$\lim_{n\to\infty}d(t_n,q)=c. \tag{12}$$

By (4) and (12), we get

$$c=\lim_{n\to\infty}\inf d(t_n,q)\leqslant\lim_{n\to\infty}\inf d(z_n,q).$$

From the above and (9), we obtain

$$\lim_{n\to\infty}d(z_n,q)=c. \tag{13}$$

By (7), (8) and (13), it shows that

$$\lim_{n\to\infty}d(x_n,z_n)=0. \tag{14}$$

Also from the inequality (CN^*), $Sq=\{q\}$ and (3), we have

$$d^2(t_n,q)=d^2(\gamma_n z_n\oplus(1-\gamma_n)w_n,q)\leqslant$$
$$\gamma_n d^2(z_n,q)+(1-\gamma_n)d^2(w_n,q)-\gamma_n(1-\gamma_n)d^2(z_n,w_n)\leqslant$$

$\gamma_n d^2(z_n,q)+(1-\gamma_n)dist^2(q,Sz_n)-\gamma_n(1-\gamma_n)d^2(z_n,w_n)\leqslant$
$\gamma_n d^2(z_n,q)+(1-\gamma_n)H^2(Sq,Sz_n)-\gamma_n(1-\gamma_n)d^2(z_n,w_n)\leqslant$
$\gamma_n d^2(z_n,q)+(1-\gamma_n)d^2(z_n,q)-\gamma_n(1-\gamma_n)d^2(z_n,w_n)\leqslant$
$d^2(x_n,q)-\gamma_n(1-\gamma_n)d^2(z_n,w_n)$ (15)

By (4) we get

$d^2(y_n,q)=d^2(\beta_n x_n\oplus(1-\beta_n)Tt_n,q)\leqslant$
$\beta_n d^2(x_n,q)+(1-\beta_n)d^2(Tt_n,q)-\beta_n(1-\beta_n)d^2(x_n,Tt_n)\leqslant$
$\beta_n d^2(x_n,q)+(1-\beta_n)d^2(t_n,q)-\beta_n(1-\beta_n)d^2(x_n,Tt_n)\leqslant$
$d^2(x_n,q)-\beta_n(1-\beta_n)d^2(x_n,Tt_n).$ (16)

Similarly, by (5) we have

$d^2(x_{n+1},q)=d^2(\alpha_n Tx_n\oplus(1-\alpha_n)Ty_n,q)\leqslant$
$\alpha_n d^2(Tx_n,q)+(1-\alpha_n)d^2(Ty_n,q)-\alpha_n(1-\alpha_n)d^2(Tx_n,Ty_n)\leqslant$
$\alpha_n d^2(x_n,q)+(1-\alpha_n)d^2(y_n,q)-\alpha_n(1-\alpha_n)d^2(Tx_n,Ty_n)\leqslant$
$d^2(x_n,q)-\alpha_n(1-\alpha_n)d^2(Tx_n,Ty_n).$ (17)

Because of $0<a\leqslant\alpha_n,\beta_n,\gamma_n\leqslant b<1$, this together (15) with (16) and (17) shows that

$$0\leqslant\gamma_n(1-\gamma_n)d^2(z_n,w_n)\leqslant d^2(x_n,q)-d^2(t_n,q)\to 0\ (n\to\infty),$$

$$0\leqslant\beta_n(1-\beta_n)d^2(x_n,Tt_n)\leqslant d^2(x_n,q)-d^2(y_n,q)\to 0\ (n\to\infty),$$

$$0\leqslant\alpha_n(1-\alpha_n)d^2(Tx_n,Ty_n)\leqslant d^2(x_n,q)-d^2(x_{n+1},q)\to 0\ (n\to\infty),$$

Indeed, from (7),(10) and (12), we obtain that

$$\lim_{n\to\infty}d(z_n,w_n)=\lim_{n\to\infty}d(x_n,Tt_n)=\lim_{n\to\infty}d(Tx_n,Ty_n)=0. \tag{18}$$

Since $t_n=\gamma_n z_n\oplus(1-\gamma_n)w_n$, we have

$$d(t_n,x_n)=d(\gamma_n z_n\oplus(1-\gamma_n)w_n,x_n)\leqslant$$
$$\gamma_n d(z_n,x_n)+(1-\gamma_n)d(w_n,x_n)\leqslant$$
$$\gamma_n d(z_n,x_n)+(1-\gamma_n)\{d(w_n,z_n)+d(z_n,x_n)\}$$
$$\to 0(n\to\infty). \tag{19}$$

By nonexpansiveness of T, and this together (18) with (19) shows that

$$d(x_n,Tx_n)\leqslant d(x_n,Tt_n)+d(Tt_n,Tx_n)+d(Tx_n,Ty_n)\leqslant$$
$$d(x_n,Tt_n)+d(t_n,x_n)+d(Tx_n,Ty_n)$$
$$\to 0(n\to\infty).$$

Immediately, we get

$$\lim_{n\to\infty}d(x_n,Tx_n)=0.$$

(iii) Because of the nonexpansiveness of S, from (14) and (18), we get

$$\operatorname{dist}(x_n,Sx_n)\leqslant d(x_n,z_n)+\operatorname{dist}(z_n,Sz_n)+H(Sz_n,Sx_n)\leqslant$$
$$d(x_n,z_n)+\operatorname{dist}(z_n,Sz_n)+d(z_n,x_n)\leqslant$$
$$2d(x_n,z_n)+d(z_n,w_n)$$
$$\to 0(n\to\infty).$$

This implies that

$$\lim_{n\to\infty}\operatorname{dist}(x_n,Sx_n)=0.$$

(iv) By virture of $\lambda_n>\lambda>0$, lemma 1 and nonexpansiveness of J_λ, and $z_n=J_{\lambda_n}x_n$, we have

$$d(x_n,J_\lambda x_n)\leqslant d(x_n,z_n)+d(z_n,J_\lambda x_n)\leqslant$$
$$d(x_n,z_n)+d(J_{\lambda_n}z_n,J_\lambda x_n)=$$
$$d(x_n,z_n)+d(J_\lambda(\frac{\lambda_n-\lambda}{\lambda_n}J_{\lambda_n}x_n\oplus\frac{\lambda}{\lambda_n}x_n),J_\lambda x_n)\leqslant$$
$$d(x_n,z_n)+\frac{\lambda_n-\lambda}{\lambda_n}d(J_{\lambda_n}x_n,x_n)+\frac{\lambda}{\lambda_n}d(x_n,x_n)=$$
$$(2-\frac{\lambda}{\lambda_n})d(x_n,z_n)\to 0(n\to\infty).$$

Hence, we get

$$\lim_{n\to\infty}d(x_n,J_\lambda x_n)=0.$$

(v) Here, we assume that the mapping S is hemi-compact. By the step (iii), we get $\lim_{n\to\infty}\operatorname{dist}(x_n,Sx_n)=0$. By the hemi-compactness of S and we have htat there exists a subsequence $\{u_n\}$ of $\{x_n\}$, which converges strongly to an element q in D. From the front (ii)-(iv), we obtain that

$$\lim_{n\to\infty}d(u_n,Tu_n)=0,\ \lim_{n\to\infty}\operatorname{dist}(u_n,Su_n)=0 \text{ and } \lim_{n\to\infty}d(u_n,J_\lambda u_n)=0.$$

It follows by the nonexpansiveness of T, and by the nonexpansiveness of J_λ that $q=Tq=J_\lambda q$, we get

$$q\in F(T)\cap F(J_\lambda)=F(T)\cap\operatorname*{argmin}_{y\in D}f(y). \tag{20}$$

By the nonexpansiveness of S, we have

$$\operatorname{dist}(q,Sq)\leqslant d(q,u_n)+\operatorname{dist}(u_n,Su_n)+H(Su_n,S_q)\leqslant$$
$$2d(q,u_n)+\operatorname{dist}(u_n,Su_n)\to 0(n\to\infty).$$

It shows that $\operatorname{dist}(q,Sq)=0$. This implies that $q\in Sq$. Therefore, we get $q\in F(S)$. By (20), we have

$$q\in F(T)\cap F(S)\cap\operatorname*{argmin}_{y\in D}f(y)=\Omega.$$

According to the double extract subsequence principle, we conclude that the sequence $\{x_n\}$ strongly converges to a point q in Ω.

Since every multi-valued mapping $S:D\to CB(D)$ is hemi-compact if D is a compact subset of X. Then the following result can be obtained from Theorem 1 immediately.

Theorem 2 Let D be a compact convex nonempty subset of a complete $CAT(0)$ space X. The sequence $\{x_n\}$ generated by algorithms (2) satisfied the conditions (2)-(4) of Theorem 1. Then the sequence $\{x_n\}$ converges strongly to a point in Ω.

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CAT(0)空间混合非扩张映射修正临近点算法的收敛性

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摘要:提出了一种修正的临近点算法,来寻找单值非扩张映射、集值非扩张映射不动点点集以及凸下半连续泛函极小元集的公共元,获得了所提出算法的收敛性。

关 键 词:*CAT*(0)空间;临近点算法;单值非扩张映射;多值非扩张映射;预解式